On the Value of Synchronous Downlink MIMO-OFDMA Systems with Linear Equalizers

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Abstract—Cellular radio systems are often limited due to the presence of cochannel interference. Basically, radio systems may be operated by utilizing asynchronous or synchronous downlink transmission from all base stations in the system, where synchronization between terminals and their serving base station is mandatory in both concepts. We provide a comparison between the theoretical achievable spectral efficiency in an orthogonal frequency division multiple access systems using different linear equalizers and their resulting performance taking estimation errors into account. It is shown, that the choice of the appropriate receiver depends on the degree of synchronization in the system. We demonstrate that a 3G Long Term Evolution radio system may achieve higher spectral efficiency thanks to interference suppression if fully synchronized data transmission from all base stations is introduced.

I. INTRODUCTION

Multi-cell interference is undoubtedly the major limiting factor in full-coverage broadband wireless access networks. Mitigating its effect onto the downlink data transmission is a key challenge in future wireless communication systems. The better the knowledge about the interference channels, the better is also the basis for any interference suppression techniques.

Existing and future wireless standards, as 802.11n, WiMAX, 3G HSDPA and its long-term evolution (3G-LTE) have built-in means for multiple-input multiple-output (MIMO) signaling. Multi-antenna terminals enable interference suppression at the receiver side and thus the radio system may profit from a higher spectral efficiency. Simple receiver-based schemes like optimal interference rejection combining [1] are known to significantly increase the system’s spectral efficiency [2]. However, its applicability in MIMO-OFDMA systems is not fully evident yet, since it is highly sensitive to channel estimation errors. This work shows the spectral efficiency gains in an OFDMA system using the interference rejection combining (IRC) instead of the maximum ratio combining (MRC) receiver, while considering channel estimation errors. We always assume synchronization between mobile terminals (MTs) and their serving base station (BS) [3]. The system parameters are chosen from the current draft specification of the 3G Long Term Evolution (3G-LTE) [4].

While the MRC (7) receiver only requires knowledge on its own channel vector, the IRC (5) approach additionally requires the system’s covariance matrix. Thus, the latter is shown to be highly sensitive to estimation errors while the first is quite robust against estimation errors. For interference suppression we consider two different techniques for covariance estimation:

First, asynchronous downlink data transmission from all BSs in the observation area is assumed, where data symbols are independent identically distributed (i.i.d.). Hence, the receive covariance may be estimated, i.e. averaged, over these symbols with a decreasing mean square error (MSE) for increasing number s of data symbols [5], [6].

Second, we assume a fully synchronized system and suggest to use a multi-cell channel estimation based on partially orthogonal virtual pilot sequences, as described in [7]. We demonstrate that interference suppression techniques increase the spectral efficiency in 3G-LTE radio system, if a fully synchronized data transmission from all BSs is introduced.

Section II describes the system model and the receiver structures which are the basis of this work. In section III the covariance estimation based on transmitted data symbols as well as on the correlation-based estimator is described. Results are given in section IV and the paper concludes in section V.

II. SYSTEM MODEL

The downlink MIMO-OFDMA transmission via \( N_T \) transmit and receive antennas each subcarrier is described by

\[
y = \mathbf{H} \mathbf{C}_i \mathbf{x} + \mathbf{n},
\]

where \( \mathbf{H} \) is the \( N_R \times N_T \) channel matrix and \( \mathbf{C}_i \) the unitary \( N_T \times N_R \) pre-coding matrix; \( \mathbf{x} \) denotes the \( N_T \times 1 \) vector of transmit symbols; \( \mathbf{y} \) and \( \mathbf{n} \) denote the \( N_R \times 1 \) vectors of the received signals and of the additive white Gaussian noise (AWGN) samples.

In the following assume that all BSs provide \( q \) fixed unitary beam sets \( \mathcal{C}_q \), \( q \in \{1, ..., Q\} \). Each beam set contains \( N_T \) fixed beams \( \mathbf{b}_{i,u}^q \in \mathcal{C}_q \) with \( u \in [1, N_T] \). Each BS \( i \) independently selects one of these sets. The BSs may select a limited number of the available beams in \( \mathcal{C}_q \) to serve the users simultaneously, yielding the set \( \mathcal{C}_i \). The transmit power per beam is then limited to \( p_i/|\mathcal{C}_i| \), where \( p_i \) is the total available power for BS \( i \) and \( |\mathcal{C}_i| \) is the cardinality of \( \mathcal{C}_i \). In this work, we use pre-coding weights...
The cellular environment is given by

\[ p_i \left( \tilde{h}_{i,u} + \delta_{i,u} \right)^H \tilde{h}_{i,u} \tilde{h}_{i,u}^H (\tilde{h}_{i,u} + \delta_{i,u}) \]

where \( \tilde{h}_{i,u} \) denotes the covariance matrix of the received signal \( y^m \), i.e. \( R_{yy} = E \left[ y^m (y^m)^H \right] \). The MMSE receiver yields a post-equalization SINR

\[ \text{SINR}_{u}^{\text{MMSE}} \geq \frac{p_i}{|C|} \left( \frac{\tilde{h}_{i,u}^H \tilde{h}_{i,u}}{\tilde{h}_{i,u}^H \tilde{h}_{i,u}} \right) \]

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For reference purpose we compare these results with results achievable by using a MRC receiver

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\[ \text{SINR}_{u}^{\text{MRC}} \geq \frac{p_i}{|C|} \left( \frac{\tilde{h}_{i,u}^H \tilde{h}_{i,u}}{\tilde{h}_{i,u}^H \tilde{h}_{i,u}} \right) \]

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yielding a post-equalization SINR bound

\[ \text{SINR}_{u}^{\text{MRC}} \geq \frac{p_i}{|C|} \left( \frac{\tilde{h}_{i,u}^H \tilde{h}_{i,u}}{\tilde{h}_{i,u}^H \tilde{h}_{i,u}} \right) \]

Introducing channel estimation errors yields to an estimate \( \hat{h}_{i,u} = h_{i,u} + \delta_{i,u} \). Here, \( \hat{h} \) denotes the estimate of variable \( x \) and \( \delta_{i,u} \) denotes the zero-mean Gaussian distributed error with variance \( \mu \), being the normalized MSE for channel estimation. Thus, the MRC receiver is given by \( \hat{w}_{u}^{\text{MRC}} = \hat{h}_{i,u} \). The post-equalization SINR at the receiver may be lower bounded by (11). The achievable SINRs from (8) degrade by the loss caused by estimation errors given by \( \mu \). Assuming \( N_R = 2 \) and a normalized channel estimation MSE \( \mu = 0.1 \) yields a maximum SINR loss of 5%.

### III. Covariance estimation

For interference suppression at the MT based on (5), we require to obtain the system’s covariance matrix \( R_{yy} \). In the following two mechanisms are considered to estimate the desired matrix.

#### A. Covariance Estimator

Knowledge on interference conditions may be obtained by estimating the covariance matrix \( R_{yy} = E \left[ yy^H \right] \) of the received signal vector \( y^m \) across several subsequently received data symbols. Therefore, asynchronous downlink transmission from all BSs in the system may be sufficient. By assuming data transmission of i.i.d. symbols \( x_k \) across
channel $k$ and averaging over $s$ symbols the estimation error in $\hat{R}_{yy}$ decreases with $s$ [5], [6]. Let the total number of transmitted data symbols across a quasi-static channel be given by $S$.

$$\hat{R}_{yy} = \frac{1}{S} \sum_{sy} \mathbb{E} \left[ \left( \sum_{y_k,l} h_{k,l} x_{k,l}(s) + n \right) \left( \sum_{y_k,l} h_{k,l} x_{k,l}(s) + n \right)^H \right]$$  \hspace{1cm} (12)

### B. Correlation-based estimator

Alternatively, we assume a pilot-aided multi-cell channel estimation. Modern cellular systems may not provide as many orthogonal pilot symbols as required to estimate all interfering downlink channels independently. In the 3G-LTE specification\(^2\), common pilot symbols, defined on orthogonal time and frequency resources, are intended for the purpose of intra-cell channel estimation. Orthogonal Zadoff-Chu sequences of length three enable the separated estimation of all three downlink channels belonging to the sectors of the same BS. However, downlink channels from other sites are fully correlated. Therefore, common pilots are shifted over frequency and additionally scrambled using pseudo-random sequences to be defined by the network operators.

Recently, a virtual pilot sequence allocation was suggested in [7] allowing for optimum correlation-based channel estimation in fully synchronized multi-cell systems, which usually suffer from inter-cell interference. The suggested concept enables mobile terminals to distinguish more of the strong interference channels with an increasing length of the correlation window utilized for the estimation process.\(^3\) It does not require higher pilot overhead than in current systems but results in a trade-off between the mobility of the user and the ability to track interfering channels. In this way, channels of nearby base stations may be estimated at an early stage due to the provided pattern, and therefore be separated using a short correlation window, e.g. over two transmission time intervals (TTIs). Estimating the channels of distant BSs may be less important due to their higher average path loss. Therefore, sequences requiring higher correlation window lengths to be orthogonal with its own sequence are intended for these cells.

However, assuming the BSs to serve their terminals with pre-coded channels results in another limitation. MTs are able to estimate the channels based on the common pilots, but they are further required to know the pre-coding weights to estimate the effective channels. This may be achieved by defining dedicated pilots, which introduces higher overhead to the radio system. Another way could be the decoding of other BS’s control channels e.g. by using ordered successive interference cancellation (SIC) techniques. In this work we limit the BSs to use a single set of pre-coding weights, i.e. $q = 1$, for downlink beamforming and thus the codebook entries are known at the MTs.

### IV. Performance Evaluation

The performance is investigated in a triple-sectored hexagonal cellular network with 19 BSs in total. The spatial channel model (SCME) with urban macro scenario parameters is used [8] yielding an equivalent user’s geometry as reported in [9]. The MTs are always served by the BS whose signal is received with highest average power over the entire frequency bandwidth and in each independent channel realization. For capacity evaluation only MTs being placed inside of the center cell will be considered, so that BS signals transmitted from 1st and 2nd tier model the inter-cell interference [2]. For simplicity, we employ the time-division multiple access (TDMA) round-robin scheduler for all MTs in the inner cell. In this case, the whole system bandwidth is assigned to the user $m \in \mathcal{M}$ for a given time slot. The same terminal is scheduled again after $|\mathcal{M}|$ time slots. The employed scheduler ensures a fair resource management for all MTs in the system. In fact, frequency-selective scheduling would increase the system throughput further [2], but also increases the complexity of the system. The basic system settings for simulations are summarized in Table I.

### Performance Evaluation

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>channel model</td>
<td>3GPP SCME(^4)</td>
</tr>
<tr>
<td>type</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>scenario</td>
<td>urban-macro</td>
</tr>
<tr>
<td>additional modifications</td>
<td>scenario-mix(^5)</td>
</tr>
<tr>
<td>$f_c$</td>
<td>2 GHz</td>
</tr>
<tr>
<td>system bandwidth</td>
<td>31.72 MHz, 128 RBs</td>
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<tr>
<td>signal bandwidth</td>
<td>18 MHz</td>
</tr>
<tr>
<td>intersite distance</td>
<td>500m</td>
</tr>
<tr>
<td>number of BSs</td>
<td>19 having 3 sectors each</td>
</tr>
<tr>
<td>antenna elements ; spacing</td>
<td>1.2 ; 4λ</td>
</tr>
<tr>
<td>transmit power</td>
<td>46 dBm</td>
</tr>
<tr>
<td>sectorization</td>
<td>triple, with FWHM(^6) of 68°</td>
</tr>
<tr>
<td>BS height</td>
<td>32m</td>
</tr>
<tr>
<td>antenna elements ; spacing</td>
<td>1.2 ; $\lambda/2$</td>
</tr>
<tr>
<td>MT height</td>
<td>2m</td>
</tr>
</tbody>
</table>

\(^2\)3GPP TS 36.211 Release 8
\(^3\)Note, users should choose their specific window sizes according to their velocity, i.e. to their coherence time of the channel.
\(^4\)Spatial Channel Model Extended.
\(^5\)i.e. each cell, consisting of 3 sectors, may have different channel conditions, e.g. line of sight (LOS) or non line of sight (NLOS).
system. For comparison three different measures are of major relevance.

Case A: The achievable spectral efficiency with perfectly known variables, i.e. equalization vector $w_i$, desired and interfering signals $\hat{h}_{i,u}$ and $z_i$, respectively.

$$\text{SINR}_u \geq p_i \frac{w_i^H \hat{h}_{i,u} \hat{h}_{i,u}^H w_i}{w_i^H Z_i w_i}$$  \hspace{1cm} (13)

Case B: The resulting spectral efficiency utilizing the estimated equalization vectors $\hat{w}_i$, i.e. the rate which may be achieved in maximum using an erroneous estimated receiver.

$$\hat{\text{SINR}}_u \geq p_i \frac{\hat{w}_i^H \hat{h}_{i,u} \hat{h}_{i,u}^H \hat{w}_i}{\hat{w}_i^H Z_i \hat{w}_i}$$  \hspace{1cm} (14)

Case C: The difference $\Delta_{\text{SINR}}$ between the estimated and achievable SINR at the MT. Where the MT utilizes the estimates $\hat{w}_i$, $\hat{h}_{i,u}$ and $Z_i$.

$$\Delta_{\text{SINR}} = p_i \frac{\hat{w}_i^H \hat{h}_{i,u} \hat{h}_{i,u}^H \hat{w}_i}{\hat{w}_i^H Z_i \hat{w}_i} - \text{SINR}_u$$  \hspace{1cm} (15)

Non-synchronized downlink transmission from all BSs: If utilizing the MRC receiver the desired signal $h_{i,u}$ needs to be estimated. From [11] we assume the estimation error to be modeled by a Gaussian distribution with a variance equal to a specific MSE defined by $\mu \in \{-10, -20, -30\}$ dB. Fig. 1(a) compares the sector spectral efficiencies using a TDMA round-robin scheduler for the single-input single-output (SISO) and MIMO $2 \times 2$ for case A. Furthermore, it may be observed that the case B performance assuming a $\mu = -10$ dB for channel estimation is sufficient to approach 99% of the MRC performance with perfect channel estimates. Thus, the lower bound for the SINRs, which was determined by the analytically estimate in (11), is quite close to the resulting performance from our simulations.

For Fig. 1(b) the covariance estimator from section III-A is employed across 1 and 2 TTIs, i.e. 7 and 14 OFDM data symbols, yielding an estimate $\hat{R}_{yy}$. In combination with the channel estimation error model, which was already introduced for MRC, it is possible to determine $\hat{w}_u^{\text{MMSE}} = p_i \hat{R}_{yy}^{-1} \hat{h}_{i,u}$. Comparing both figures it may be observed that the achievable performance of the IRC receiver highly depends on the estimate $\hat{h}_{i,u}$ showing hardly any gain for a $\mu = -20$ dB compared to the MRC approach. In case of an assumed $\mu = -30$ dB and a covariance estimation over 2 TTIs, the system outperforms the simple MRC receiver in 60% of all cases. However, these assumptions are not feasible in a system under realistic conditions. Thus, it is doubtful whether these IRC gains are still present.

Synchronized downlink transmission from all BSs: Fig. 2 compares case A and B for the correlation-based estimator, described in section III-B. Assuming a multi-cell channel estimation over 0.5 TTIs, i.e. only one 3G-LTE slot, clearly outperforms the MRC receiver. Thus, 3G-LTE systems may already profit from higher spectral efficiencies when employing IRC using the correlation-based estimator. Therefore, any additional scrambling for common pilots symbols of different sites has to be introduced, but downlink transmission from all BSs must be synchronized. In this case the achievable median spectral efficiency reaches 94.7% of the case A performance. If the estimation is done over 1 TTI the median performance reaches 98.4% of case A. Finally, after 2 TTIs there is hardly any difference between the performance of case A and B.

Furthermore it can be observed that estimation errors caused by the correlation-based estimator only result in a constant shift of the original cumulative distribution function (cdf) with perfect channel knowledge and IRC (Fig. 2). In contrast, the estimation errors caused by the covariance estimator dramatically change the shape of the distribution, removing high spectral efficiencies of the system which indicates the highly error limited behavior (Fig. 1(b)).

Fig. 3 compares the resulting $\Delta_{\text{SINR}}$ for asynchronous and synchronous downlink transmission from all BSs in
the MMSE receiver for interference suppression at the ter-
achievability of SINR is maximized. Hence, we suggest to use
from linear equalizers, which minimize the MSE while the
fully synchronized transmission system, one may benefit
but with relatively high standard deviation. Assuming a
that MTs utilizing MRC receivers tend to underestimate
for MSE

\[ \mu = -10 \text{ dB} \]

is sufficient to approach the upper bound
given by perfect channel knowledge at the receiver. Interfer-
ence suppression requires channel estimates of higher
precision and therefore does not seem to be feasible in
non-synchronized systems, e.g. by employing covariance
estimation based on independent OFDM data symbols.
Including the gains of interference suppression into cellular
systems requires more precise multi-cell channel know-
ledge. This may be enabled by introducing synchronized
downlink transmission and a multi-cell channel estimation
as suggested in [7]. We demonstrated that a 3G-LTE
radio system achieves 94.7% of the spectral efficiency
available with perfect channel knowledge and IRC, if fully
synchronized data transmission from all base stations is
introduced. Improving the knowledge of the interfering
channels results in a performance close to the optimum of
linear equalizers and a decreasing estimation error \( \Delta_{\text{SINR}} \)
between the estimated and the achievable SINRs.

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