Joint Power Control, Beamforming and BS Assignment for Optimal SIR Assignment

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***Abstract*—This paper considers the open problem of joint power control, beamforming, and base station assignment to achieve an optimal SIR assignment in uplink cellular networks. The adaptive beamforming and adaptive base station assignment strategies in addition to power control will broaden the feasible SIR region. The problem is challenging, since the feasible SIR region is the union of regions from all possible combinations of beamformer and base station assignment, which is generally not log-convex. By using concave interference function property, the Pareto boundary of the feasible SIR region of joint power control, beamforming, and base station assignment is characterized. Since the optimal SIR assignment solution must lie on the Pareto boundary of the feasible SIR region to achieve the highest possible efficiency, the optimization problem can be reduced to the optimization along the Pareto optimal boundary. Thus instead of searching the operating point over the whole feasible SIR region, a distributed greedy algorithm that searches a possible optimal solution on the Pareto boundary is developed. From the simulation, we show that the performance gain obtained from joint power control, beamforming, and base station assignment is quite significant.**

I. Introduction

The wireless channel has a broadcast nature, therefore the transmission signal from one user will interfere the transmission signal of other users and degrades the performance considerably. In order to support high Quality of Service (QoS) for each user, it is required to control the multiuser interference. The QoS metric that is considered in this paper is the data rate. Power control has been considered as an important mechanism to control the Signal-to-Interference Ratios (SIR), which has a direct relationship with QoS. The higher the SIR, the better the QoS one user can meet.

The conventional distributed power control algorithm tries to minimize the overall transmit power while maintaining certain fixed SIR requirement (see, for instance [1] [2] [3] and references therein). The algorithm was motivated for wireless voice networks, namely to satisfy the minimum SIR requirement, since the perception of voice quality by human ear remains the same beyond a certain SIR. However, in wireless data network the operator would like to support variable SIR assignment, where higher SIR provides a user with higher data rate, while smaller SIR can still support lower data rate applications, such as E-mail, SMS and voice. Due to multiuser interference, unfortunately not all SIR vectors can be supported. The feasible SIR region is defined as the region, in which any SIR assignment for all users can be concurrently supported. The SIR assignment can not go beyond the feasible SIR region. Thus it is important to find a distributed algorithm to assign SIRs that belong to the feasible SIR region and yet optimal. Distributed power control algorithm, which achieves the optimal SIR assignment, has been recently found [4]. A survey on state of the art of power control algorithms and its open problems is presented in [5].

If the base station is equipped with multiple receive antennas, additional degrees of freedom can be beneficially exploited by dynamically adjusting the receive beamforming vectors and transmit power jointly. The additional degrees of freedom will broaden the feasible SIR region and provide a higher system throughput. Adaptive base station assignment further increases the degrees of freedom as now macro diversity can be exploited. Thus, the combination of adaptive beamforming and adaptive base station assignment will again broaden the feasible SIR region. Joint power control with beamforming and base station assignment has been considered in [6]. As their objective is to minimize total transmit power with fixed SIR assignment, the algorithm doesn't fully exploit the knowledge of the broader feasible SIR region due to adaptive beamforming and adaptive base station assignment. The knowledge of feasibility allows the network operator to push the network towards higher efficiency and concurrently prevents the network from collapsing. The authors in [7] proposed a centralized algorithm to solve the SIR balancing problem of joint power control with beamforming and base station assignment. SIR balancing has the objective to achieve max-min fairness, where all users are treated as fairly as possible by making their SIRs as equal as possible. However, this criteria results in a poor throughput performance of a cellular system due to the large spread of different users SIR-conditions.

This paper extends the distributed power control algorithm in [4] to the case where adaptive beamforming and adaptive base station assignment are jointly optimized with power control to maximize a class of utility functions introduced in [8]. While the feasible SIR region for power control has a log-convexity property [9], which allows convex optimization problem formulation and ensuring global optimal solution, unfortunately the feasible SIR region for power control in combination with adaptive beamforming and adaptive base station assignment is generally not log-convex. Thus multiple local optimum solutions may exist. Given a feasible SIR

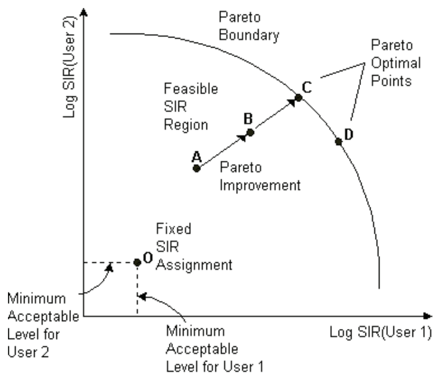


Fig. 1. Illustration of feasible SIR region for 2 users.

region, a change from one assignment to another that can make at least one user SIR better off without making any other individual SIR worse off is called a Pareto improvement (see Fig. 1: $A \rightarrow B, B \rightarrow C$). An SIR assignment is Pareto optimal when no further Pareto improvements can be made (e.g., C or D in Fig. 1). The Pareto optimal points of a feasible SIR region form the Pareto-optimal boundary. The optimal SIR assignment solution must lie on the Pareto-optimal boundary of the feasible SIR region to achieve the highest possible efficiency. Fortunately, the boundary of the feasible SIR region of the joint power control, beamforming and base station assignment can be characterized by using the property of concave interference function [10]. Therefore the optimization problem over the whole feasible region reduces to the optimization along the Pareto-optimal boundary. Thus instead of searching the operating point over the whole feasible SIR region, a distributed greedy algorithm that searches a possible optimal solution on the Pareto-optimal boundary is developed. From the simulation, we show that each addition of adaptive receive strategies, such as adaptive beamforming and adaptive base station assignment, will improve the QoS of the network.

II. System Model and Problem Formulation

Consider the uplink of a wireless cellular network with M mobile stations, each equipped with 1 transmit antenna, sending independent information to N base stations, where each base station is equipped with K receive antennas. The signal transmitted by mobile station i is received by its serving base station π_i. Denote $\mathbf{h}_{\pi_i i}$ as the $K \times 1$ channel gain vector from mobile station i to its serving base stations π_i. Let x_i be the information signal sent by mobile i. The receiver employs a set of linear beamforming vectors $\mathbf{w}_i \in \mathcal{C}^{K\times 1}$, with $||\mathbf{w}_i||_2 = 1$, resulting in a received signal for mobile station i given by

$$\mathbf{w}_i^H \mathbf{y}_{\pi_i i} = \underbrace{(\mathbf{w}_i^H \mathbf{h}_{\pi_i i}) x_i}_{\text{Useful signal for MS } i} + \underbrace{\sum_{j \neq i} (\mathbf{w}_i^H \mathbf{h}_{\pi_i j}) x_j}_{\text{Interference from another MS}} + \underbrace{\mathbf{w}_i^H \mathbf{z}_{\pi_i}}_{\text{Noise}}, \tag{1}$$

where $\mathbf{z}_{\pi_i}$ is an additive $K \times 1$ Gaussian noise with variance σ^2. Let $E\{|x_i|^2\} = p_i$ be the transmit power from mobile i. The transmit power from all users are collected in a power allocation vector, $\mathbf{p} = [p_1, \ldots, p_M]$. The instantaneous SIR, γ_i, for link i is

$$\gamma_i = \frac{p_i |\mathbf{w}_i^H \mathbf{h}_{\pi_i i}|^2}{\sum_{j\neq i} p_j |\mathbf{w}_i^H \mathbf{h}_{\pi_i j}|^2 + \sigma^2}. \tag{2}$$

The interference coupling matrix is defined as in [7]

$$[\mathbf{V}(\mathbf{W}, \boldsymbol{\pi})]_{ij} = \begin{cases} \frac{\mathbf{w}_i^H \mathbf{R}_{\pi_i j} \mathbf{w}_i}{\mathbf{w}_i^H \mathbf{R}_{\pi_i i} \mathbf{w}_i}, & \text{if } i \neq j, \\ 0, & \text{if } i = j, \end{cases} \tag{3}$$

where the matrix $\mathbf{R}_{\pi_i j} = E[\mathbf{h}_{\pi_i j} \mathbf{h}_{\pi_i j}^H]$ is the spatial covariance matrix of the vector channel $\mathbf{h}_{\pi_i j}$. The interference coupling matrix, $\mathbf{V}(\mathbf{W}, \boldsymbol{\pi})$, contains interference coefficients, which determine in which way the users are affected by interference. It depends on the receive beamforming vectors strategy $\mathbf{w}_1, \ldots, \mathbf{w}_M$, which can be collected in a matrix $\mathbf{W}$, and the base stations assignment $\pi_1, \ldots, \pi_M$, which can be collected in a vector $\boldsymbol{\pi}$. The interference experienced by the ith user can be modeled by an interference function, which is defined as

$$I_i(\mathbf{p}, \mathbf{W}, \boldsymbol{\pi}) = [\mathbf{V}(\mathbf{W}, \boldsymbol{\pi})\mathbf{p}]_i + c_i, \quad i = 1, \ldots, M, \tag{4}$$

with $c_i = \frac{\sigma^2}{\mathbf{w}_i^H \mathbf{R}_{\pi_i i} \mathbf{w}_i}$. Stacking the interference function in a vector, we can write

$$I(\mathbf{p}, \mathbf{W}, \boldsymbol{\pi}) = \mathbf{V}(\mathbf{W}, \boldsymbol{\pi})\mathbf{p} + \mathbf{c}. \tag{5}$$

Let γ_i be the SIR achieved by link i with $\gamma_i = p_i / I_i(\mathbf{p}, \mathbf{W}, \boldsymbol{\pi})$, we get the following equation

$$\mathbf{p} = \boldsymbol{\Gamma}(\gamma) I(\mathbf{p}, \mathbf{W}, \boldsymbol{\pi}), \tag{6}$$

where $\boldsymbol{\Gamma}(\boldsymbol{\gamma}) = diag(\gamma_1, \ldots, \gamma_M)$. Combining (5) and (6), we get

$$\mathbf{p} = (\mathbf{I} - \boldsymbol{\Gamma}(\gamma)\mathbf{V}(\mathbf{W}, \boldsymbol{\pi}))^{-1} \boldsymbol{\Gamma}(\gamma)\mathbf{c}. \tag{7}$$

To have a positive power allocation, $\mathbf{p} \succeq 0$, it is required that $(\mathbf{I} - \boldsymbol{\Gamma}(\gamma)\mathbf{V}(\mathbf{W}, \boldsymbol{\pi}))^{-1}$ to be non-negative. Thus not all SIR vectors $\boldsymbol{\gamma}$ are achievable. By the Perron-Frobenius theorem, an SIR vector $\boldsymbol{\Gamma}(\boldsymbol{\gamma})$ is feasible if and only if $\rho(\boldsymbol{\Gamma}(\gamma)\mathbf{V}(\mathbf{W}, \boldsymbol{\pi})) < 1$, where $\rho(.)$ is a spectral radius function [1]. Thus the unconstrained feasible SIR region $\boldsymbol{\gamma}$ is

$$\mathbf{B}(\mathbf{W}, \boldsymbol{\pi}) = \{\boldsymbol{\gamma} \succeq 0 : \rho(\boldsymbol{\Gamma}(\gamma)\mathbf{V}(\mathbf{W}, \boldsymbol{\pi})) < 1\}. \tag{8}$$

If the interference coupling matrix $\mathbf{V}$ doesn't depend on other variables, it is known that the logarithm of the feasible SIR region is convex [9]. However since now the interference coupling matrix $\mathbf{V}(\mathbf{W}, \boldsymbol{\pi})$ depends on beamforming strategy $\mathbf{W}$ and base station assignment strategy $\boldsymbol{\pi}$ then the feasible SIR region $\mathbf{B}(\mathbf{W}, \boldsymbol{\pi})$ becomes the union of regions associated with all possible combinations of $\mathbf{W}$ and $\boldsymbol{\pi}$. Generally the union of convex regions is not a convex region. Taking the maximum transmit power constraint $\mathbf{p}^m$ into account, the power allocation must satisfy $\mathbf{p} \preceq \mathbf{p}^m$. The power-constrained feasible SIR region is defined as

$$\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi}) = \{\boldsymbol{\gamma} \in \mathbf{B}(\mathbf{W}, \boldsymbol{\pi}) \mid \mathbf{p}(\boldsymbol{\gamma}) \preceq \mathbf{p}^m\} \qquad (9)$$

The joint power control, beamforming, and base station assignment for optimal uplink SIR subject to transmit power constraint can be formulated as a network utility maximization (NUM) problem

$$\begin{aligned} max. \quad &: \quad \sum_{i=1}^{M} U_i(\beta(\gamma_i)) \\ s.t. \quad &: \quad \boldsymbol{\gamma} \in \mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi}) \\ & \quad\quad \mathbf{p} \succeq \mathbf{0} \\ & \quad\quad \pi_i \in \mathbf{S}_i, \quad i = 1, 2, \ldots, M \\ variables \quad &: \quad \mathbf{p}, \mathbf{W}, \boldsymbol{\pi}, \boldsymbol{\gamma}, \end{aligned} \qquad (10)$$

where $\mathbf{S}_i$ is a an integer set of feasible base station assignment for mobile station i. The utility function $U_i(\beta_i)$ represents gain to the network of allocating mobile station i a quality of service (QoS) metric β_i, where $\beta_i = \beta(\gamma_i)$. The QoS metric that we consider is the capacity, given by $\beta_i = log_2(1+\gamma_i)$. A class of utility functions is introduced in [8], where different optimization goals can be achieved by varying the parameter $\alpha \geq 1$

$$U_i(\beta_i) = \begin{cases} log(\beta_i) & \text{if } \alpha = 1, \\ \frac{\beta_i^{1-\alpha}}{1-\alpha} & \text{if } \alpha > 1. \end{cases} \qquad (11)$$

The utility function $U_i(\beta_i(\gamma_i))$ for $\alpha \geq 1$ has the following properties: strictly increasing, twice differentiable, strictly concave in $\log \gamma_i$, and fair since it avoids QoS starvation of users, as $\gamma_i \rightarrow 0$ results $U_i(\beta(\gamma_i)) \rightarrow -\infty$. Proportional fairness is achieved for $\alpha = 1$, since it maximizes the sum of all the logarithmic utility functions [8]. Max-min fairness can be approximated as $\alpha \rightarrow \infty$. In this paper we consider proportional fairness utility with $\alpha = 1$ to create a balance between maximizing total wireless network throughput while at the same time allowing all users at least a minimal level of service.

Problem (10) is a joint optimization over power $\mathbf{p}$, beamforming $\mathbf{W}$, base station assignment $\boldsymbol{\pi}$ and SIR $\boldsymbol{\gamma}$. The feasibility region, $\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi})$ is generally not log-convex and globally coupled. Thus a global optimal solution is difficult to be obtained, because there may exist multiple local optimum solutions. Utilizing the so called concave interference function [10], we approximate the solution in a greedy manner, where the search of the possible optimal solution is done solely on the Pareto boundary of the feasible SIR region, denoted as $\boldsymbol{\partial}\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi})$.

III. Optimization Along Pareto-Optimal Boundary

The interference coupling matrix, $\mathbf{V}(\mathbf{W}, \boldsymbol{\pi})$, depends on the adaptive receive beamforming vectors and the adaptive base station assignment vector. Both, the choice of base station assignment π_i as well as the choice of the receive beamformer $\mathbf{w}_i$ for user i does not influence the interference perceived by other users. Thus, we can optimize the assignment independently and get a concave interference function [10]

$$\begin{aligned} I_i^{opt}(\mathbf{p}) &= \min_{\pi_i \in \mathbf{S}_i} \left(\min_{\mathbf{w}_i : ||\mathbf{w}_i||=1} [\mathbf{V}(\mathbf{W}, \boldsymbol{\pi})\mathbf{p}]_i + \frac{\sigma^2}{\mathbf{w}_i^H \mathbf{R}_{\pi_i i} \mathbf{w}_i} \right) \\ &= \min_{\pi_i \in \mathbf{S}_i} \left(\min_{\mathbf{w}_i : ||\mathbf{w}_i||=1} \frac{\mathbf{w}_i^H (\sum_{j \neq i} p_j \mathbf{R}_{\pi_i j} + \sigma^2 \mathbf{I}) \mathbf{w}_i}{\mathbf{w}_i^H \mathbf{R}_{\pi_i i} \mathbf{w}_i} \right) \\ &= \min_{\pi_i \in \mathbf{S}_i} \frac{1}{\mathbf{h}_{\pi_i i}^H \left(\sigma^2 \mathbf{I} + \sum_{j \neq i} p_j \mathbf{h}_{\pi_i j} \mathbf{h}_{\pi_i j}^H \right)^{-1} \mathbf{h}_{\pi_i i}} \end{aligned} \qquad (12)$$

Since minimization preserves concavity, $I_i^{opt}(\mathbf{p})$ is a concave interference function and belongs to the standard interference function [3], which fulfills the following axioms:

- **A1** Positivity $\;I_i(\mathbf{p}) > 0$.
- **A2** Monotonicity if $\;\mathbf{p} \geq \mathbf{p}'$, then $\;I_i(\mathbf{p}) \geq I_i(\mathbf{p}')$.
- **A3** Scalability $\;\alpha I_i(\mathbf{p}) \geq I_i(\alpha \mathbf{p})$ if $\alpha > 1$.

The concave interference function, $I_i^{opt}(\mathbf{p})$, attains the minimum interference among all choices of feasible base station assignment and beamforming vector at power allocation $\mathbf{p}$. The corresponding optimal linear beamformer for $I_i^{opt}(\mathbf{p})$ for (12) is the Minimum Mean Square Error (MMSE) beamformer (normalized to unit norm) [11]

$$\mathbf{w}_i^{opt}(\mathbf{p}) = \left(\sum_{j \neq i} p_j \mathbf{h}_{\pi_i j} \mathbf{h}_{\pi_i j}^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{h}_{\pi_i i}. \qquad (13)$$

The optimal choice of base station assignment corresponds to $I_i^{opt}(\mathbf{p})$ for (12) is given by

$$\pi_i^{opt}(\mathbf{p}) = \arg \min_{\pi_i \in \mathbf{S}_i} \left([\mathbf{V}(\mathbf{W}^{opt}, \boldsymbol{\pi})\mathbf{p}]_i + \frac{\sigma^2}{\mathbf{w}_i^{opt^H} \mathbf{R}_{\pi_i i} \mathbf{w}_i^{opt}} \right). \qquad (14)$$

Theorem 1: ***Pareto-Optimal Boundary Characterization*** Let $\boldsymbol{\partial}\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi})$ be the Pareto-optimal boundary of the feasible SIR region with adaptive beamforming and adaptive base station assignment, then the interference $I(\mathbf{p}, \mathbf{W}, \boldsymbol{\pi})$ on $\boldsymbol{\partial}\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi})$ must be the concave interference function $I^{opt}(\hat{\mathbf{p}})$ with $\hat{\mathbf{p}} \in \{\mathbf{p} \preceq \mathbf{p}^m \mid \exists i : p_i = p_i^m\}$.

Proof: The power allocation vector on the Pareto-optimal boundary must be in the set $\{\mathbf{p} \preceq \mathbf{p}^m \mid \exists i : p_i = p_i^m\}$, which is proved in [4]. Suppose the tupple $(\hat{\mathbf{p}}, \mathbf{W}', \boldsymbol{\pi}')$ is the power-, beamforming-, and BS assignment vector on Pareto-optimal boundary, so that $\boldsymbol{\gamma}' \in \boldsymbol{\partial}\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi})$ with interference $I'(\hat{\mathbf{p}})$. Assume that the pair $(\mathbf{W}', \boldsymbol{\pi}') \neq (\mathbf{W}^{opt}, \boldsymbol{\pi}^{opt})$. From equation (12) it is evident that $I^{opt}(\hat{\mathbf{p}}) \preceq I'(\hat{\mathbf{p}})$, which means that $\boldsymbol{\gamma}^{opt} \succeq \boldsymbol{\gamma}'$. Thus proving the claim by contradiction. ■

The optimal SIR assignment solution must lie on the Pareto-optimal boundary of the feasible SIR region to achieve the highest possible efficiency. Thus by utilizing the concave interference function $I^{opt}(\mathbf{p})$ from equation (12), the optimization problem (10), which is over the whole feasible SIR region

$\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi})$ reduces to the optimization along the Pareto-optimal boundary $\boldsymbol{\partial}\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi})$:

$$\begin{aligned} max. \quad &: \quad \sum_{i=1}^{M} U_i(\gamma_i) \\ s.t. \quad &: \quad p_i \geq \gamma_i I_i^{opt}(\mathbf{p}), \quad i = 1, 2, \ldots, M \\ &\quad\;\; \mathbf{0} \preceq \mathbf{p} \preceq \mathbf{p}^m \\ &\quad\;\; \pi_i \in \mathbf{S}_i, \quad i = 1, 2, \ldots, M \\ variables \quad &: \quad \mathbf{p}, \mathbf{W}, \boldsymbol{\pi}, \boldsymbol{\gamma}. \end{aligned} \tag{15}$$

If each mobile station can be connected to any base station in the network $\mathbf{S}_i = \{1, \ldots, N\}$ $\forall i$ then the feasible SIR region of the adaptive BS assignment is the union of regions associated with N^M possible combinations of BS assignment by assuming optimal adaptive beamformer (MMSE). Thus there may exist multiple local optimum points on the boundary of the feasibility region. The global optimal solution can be achieved, if we compare the optimal solution for all possible combination of base station assignment, which has the exponential complexity N^M. Therefore it is not tractable for high number of base stations N and mobile stations M. We combine the ascent-direction algorithm from [4] and the concave interference function to develop a greedy algorithm, which searches a local optimum solution on the Pareto boundary of the feasible SIR region. Ascent-direction means that $\sum_{i=1}^{M} U_i(\gamma_i[t+1]) \geq \sum_{i=1}^{M} U_i(\gamma_i[t])$, where t is the iteration index. The power-interference representation is re-parameterized into the so-called load-spillage representation, so that the SIR is re-parameterized into a load-spillage term

$$\gamma_i[t] = l_i[t]/s_i[t]. \tag{16}$$

Spillage s_i represents the effect of interference emitted by mobile station i transmission on other users in the network weighted by other user loads l_j plus the power price ν_i.

$$s_i[t] = \sum_j \left[\mathbf{V}(\mathbf{W}^{opt}, \boldsymbol{\pi}^{opt})[t]\right]_{ji} l_j[t] + \nu_i[t]. \tag{17}$$

A mobile station with large spillage would easily carry over interference into another mobile stations and hence should be assigned a smaller SIR. Link i, with an SIR γ_i, and responsible for spillage s_i, loads the network with $l_i = s_i\gamma_i$ in the sense that it is less tolerant by a factor of l_i to interference from other users in the network. The power price ν_i for mobile station i is updated depending upon the transmit power constraint p_i^m and is determined through an iterative subgradient update to bring down the SIR assignment from unconstrained feasible SIR region $\mathbf{B}(\mathbf{W}, \boldsymbol{\pi})$ to power constrained feasible SIR region $\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi})$.

$$\nu_i[t+1] = \left[\nu_i[t] + \delta_{\nu o}(p_i[t] - p_i^m)\right]^+, \tag{18}$$

where $\delta_{\nu o} > 0$, is a sufficiently-small step-size. The term $[z]^+ = \max\{z, 0\}$ denotes the projection onto the non-negative orthant. The load factor is chosen in greedy manner, where in each step the search direction is towards the largest load each user can have by utilizing concave interference function

$$l_i[t+1] = l_i[t] + \delta_l \triangle l_i[t+1] \tag{19}$$

$$\triangle l_i[t+1] = \frac{U_i'(\gamma_i[t])\gamma_i[t]}{I_i^{opt}(\mathbf{p}[t+1])} - l_i[t], \tag{20}$$

where $\delta_l > 0$ is a sufficiently small step-size for the load factor. The larger the load factor user i has, the more unsusceptible is user i against interference from other users. The ascent-direction update arrives on the Pareto-optimal boundary after it satisfies Theorem 1, namely as $\mathbf{p}[t] \in \{\mathbf{p} \preceq \mathbf{p}^m \mid \exists i : p_i = p_i^m\}$ and the current interference $I(\mathbf{p}[t], \mathbf{W}, \boldsymbol{\pi}) = I^{opt}(\mathbf{p}[t])$. Then it slides along the Pareto-optimal boundary in ascent-direction and converges towards the local utility-optimal point.

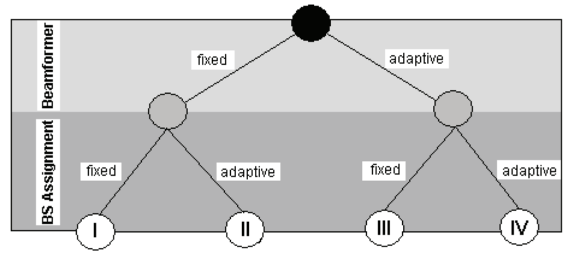


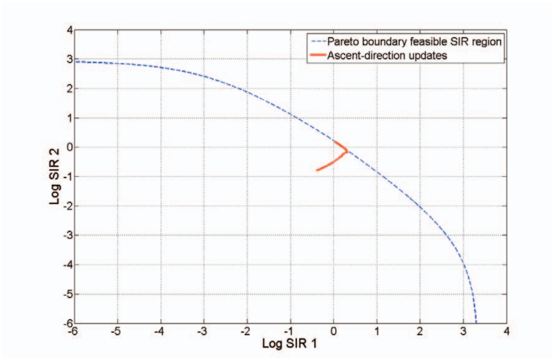
Fig. 2. The four combination possibilities.

IV. Simulation Results

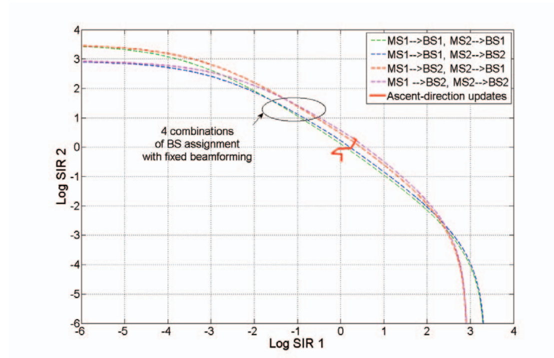
The system described in previous sections features full adaptive receive strategies: adaptive beamforming and adaptive base station assignment. Compared to a fixed scheme, each of the possible combinations of the two schemes (shown in Fig. 2) improves the system performance while causing some additional computational and communication overheads. The other schemes of the receive strategy are special cases of the full adaptive strategy algorithm, where the beamformer or the base station assignment or both of them are kept fixed. We choose Maximum Ratio Combining (MRC) beamformer as our fixed beamformer with $\mathbf{w}_i^{MRC} = \mathbf{h}_{\pi_i i}$. In fixed base station assignment case, the mobile station is connected to its closest base station.

A. 2 Cells Network

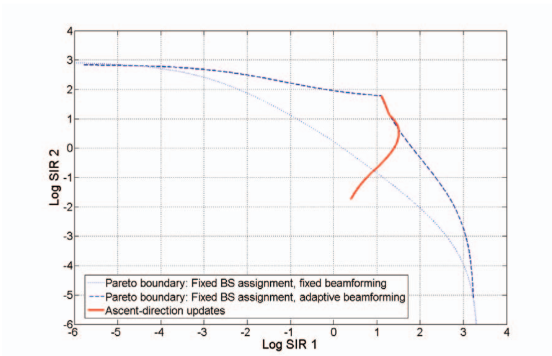
To gain insights how the feasibility SIR region is broadened with addition of each adaptive receive strategy, we plot the feasibility SIR region of the simplest uplink multicell scenario with 2 cells, where each cell consists of one base station and one mobile station. For fixed beamformer and fixed base station assignment in case I (see Fig. 3(a)), we have the most simplified version of the problem (10), where the interference coupling matrix contains a fixed constant link $\mathbf{V}$, i.e. the interference coupling matrix $\mathbf{V}$ doesn't depend on other variables. Since the objective function is log-concave and the feasible SIR region for case I $\mathbf{F}(\mathbf{p}^m)$ is log-convex, therefore this is a convex optimization problem and solved in [4]. We can see in Fig. 3(a) that the ascent-direction



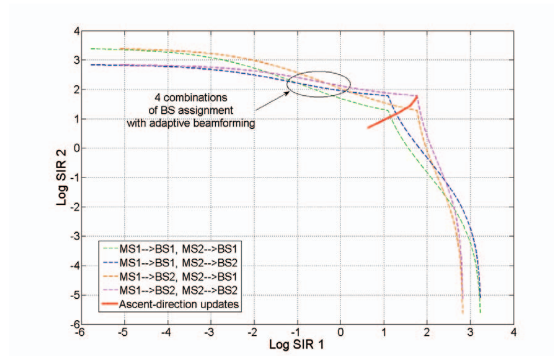
(a) Case I:Fixed beamforming and Fixed BS assignment



(b) Case II: Adaptive BS assignment and fixed beamforming



(c) Case III: Fixed BS assignment and adaptive beamforming



(d) Case IV: Adaptive BS assignment and adaptive beamforming

Fig. 3. The feasibility SIR region of the simplest uplink multicell scenario with 2 base stations and 2 mobile stations.

update slides along the Pareto-optimal boundary of the feasible SIR region $\partial\mathbf{F}(\mathbf{p}^m)$ and converges towards the utility-optimal point. In the second case, we deals with a system that features power control in combination with adaptive base station assignment. The beamforming vectors are assumed to be fixed. The interference coupling matrix $\mathbf{V}(\boldsymbol{\pi})$ depends on the base station assignment vector $\boldsymbol{\pi}(\mathbf{p})$. The SIR feasible region with adaptive BS assignment is the union of regions associated with 4 possible combinations of BS assignment by assuming fixed beamforming. The resulting region is not log-convex. The feasible SIR region $\mathbf{F}(\mathbf{p}^m, \boldsymbol{\pi})$ in case II is broader than $\mathbf{F}(\mathbf{p}^m)$ from case I. The greedy algorithm achieves a higher utility optimal point than case 1 as now MS1 is connected to BS2 and MS2 is connected to BS2 (see Fig. 3(b)). Case III equals case I in the fixed base station assignment, namely MS1 is connected to BS1 and MS2 is connected to BS2, yet instead of using a fixed beamformer, the system in case III dynamically adapts the beamformer vectors according to the current interference situation $\mathbf{W(p)}$. In case of adaptive beamformer and fixed BS assignment, the interference coupling matrix $\mathbf{V}(\mathbf{W})$ depends on adaptive receive beamforming vectors. The feasible SIR region by using optimal adaptive beamforming (MMSE) is not log-convex (see Fig. 3(c)). The region $\mathbf{F}(\mathbf{p}^m, \mathbf{W})$ is broader than the fixed beamforming case $\mathbf{F}(\mathbf{p}^m)$. The algorithm iteratively updates the SIR assignment in ascent-direction until it finds a utility optimal point. The fourth case combines both adaptive schemes. Hence the interference coupling matrix, $\mathbf{V}(\mathbf{W}, \boldsymbol{\pi})$, depends on the adaptive receive beamforming vectors and the adaptive base station assignment vector. The feasible SIR region $\mathbf{F}(\mathbf{p}^m, \mathbf{W}, \boldsymbol{\pi})$ is the union of 4 possible combinations of base station assignment by assuming optimal adaptive beamformer (MMSE). The utility optimal point is the highest among all other cases (see Fig. 3(d)).

B. System-Level Simulation in 37 Cells Network

In order to obtain the system-level result, we evaluate the algorithm in a cellular network by using the 3GPP Spatial Channel Model Extended (SCME) channel model [12], resulting in frequency flat SIMO channels (i.e., only 1 sub-carrier is considered) with omni-directional base station antennas. The simulation assumptions are given in the Table I. The maximum transmit power for each mobile station is 1 mW. As a measure of the QoS of the network, the exponent of the proportional fairness utility over $\beta_i(\gamma_i)$ is computed which is equal to the geometric mean of the user QoS

$$\prod_{i=1}^{M} \beta_i(\gamma_i)^{1/M} = \exp\left(\frac{1}{M}\sum_{i=1}^{M} \log \beta_i(\gamma_i)\right). \tag{21}$$

Figure 4 shows the convergence behavior of the algorithms for the 4 cases described before. Due to ascent-direction updates

TABLE I
SIMULATION ASSUMPTIONS.

channel model	3GPP SCME
scenario	urban-macro
f_c	2 GHz
intersite distance	500 m
number of BS	37
BS height	32 m
number of receive antenna each	2
number of MS in each cell	1
MS height	2 m
number of transmit antenna each	1

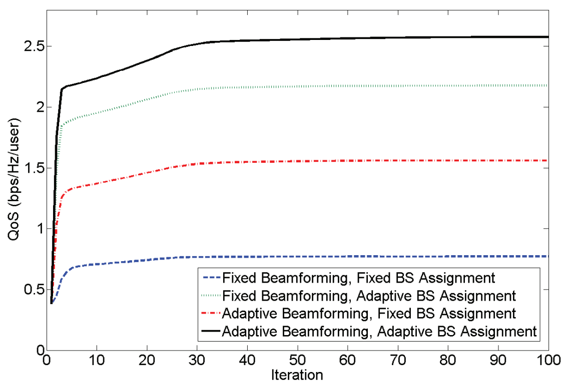


Fig. 4. Convergence behavior of the algorithm for different receive strategies.

of the algorithm, the QoS of the network increases with each iteration step until it reaches a convergence state. By each addition of adaptive receive strategy the QoS of the network will increase. The lowest QoS is achieved by using optimal power control with fixed beamforming (MRC) and fixed base station assignment with 0.77 bps/Hz/user. Using the same system setup, the joint optimal power control and optimal adaptive beamforming (MMSE) with fixed base station assignment increases the network QoS up to 1.56 bps/Hz/user. If the mobiles select its best base station which minimizes the interference function, while keeping the beamformer to be fixed , the QoS of the network increases further to 2.18 bps/Hz/user. The highest network QoS is achieved if joint optimal power control with adaptive beamforming and adaptive base station assignment is utilized with 2.58 bps/Hz/user. Figure 5 shows the cumulative distribution of the user throughput over 500 channel realizations. It can be seen that each addition of adaptive receive strategy increases the user throughput as well.

V. CONCLUSIONS

We consider adaptive beamforming and adaptive base station assignment, in addition to power control to achieve optimal SIR assignment. We characterized the Pareto-optimal boundary of the feasible SIR region of joint power control,

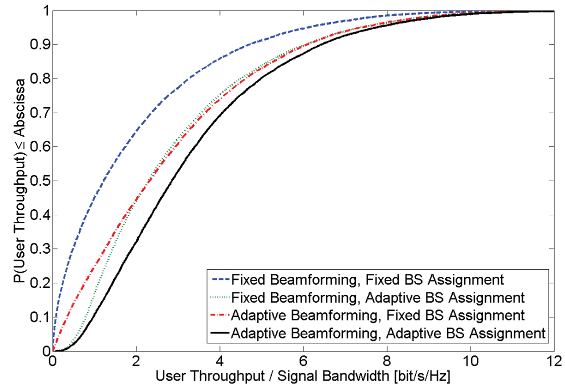


Fig. 5. User throughput distribution.

beamforming and base station assignment by utilizing the property of concave interference function. We develop a distributed greedy algorithm, where the solution is guaranteed to be on the Pareto-optimal boundary. It has been shown that, by using adaptive beamforming in combination with adaptive base station assignment, the algorithm will improve the QoS of the networks and user throughput as well.

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